(L.03)

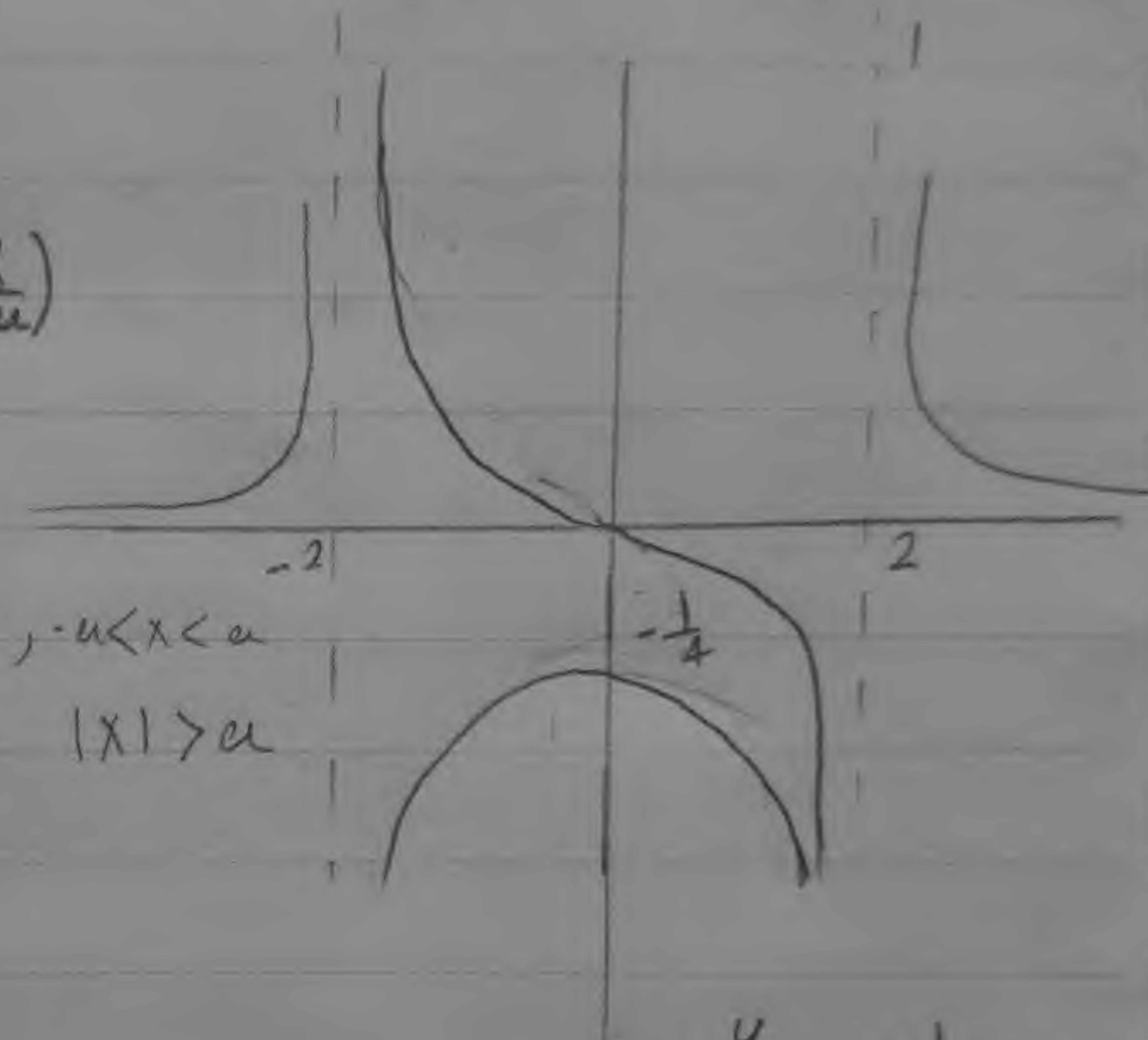
Integration by using Partial practions

Remember

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$x \neq \pm a$ $\displaystyle\int \frac{1}{a^2-x^2}\,dx =$

- $\rightarrow \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$, $-a<x<a$
- $\rightarrow \frac{-1}{a}\coth^{-1}\left(\frac{x}{a}\right)$ $|x|>a$



$y = \frac{-1}{2^2-x^2}$

$$\int \frac{u'}{u}\,dx = \ln|u|$$

$$\frac{P(x)}{Q(x)} = x^3 - x + 4$$

$$= P_x + \frac{R(x)}{Q(x)}$$

find $\int \frac{1}{a^2 - x^2} \, dx$

$$F(x) = \frac{1}{a^2 - x^2} = \frac{1}{(a-x)(a+x)} = \frac{A}{a-x} + \frac{B}{a+x}$$

$$\frac{1}{a^2 - x^2} = \frac{A(a+x) + B(a-x)}{(a-x)(a+x)}$$

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$$Aa + Ax + Ba - Bx = 1$$

$$a(A+B) + x(A-B) = 1$$

$$A - B = 0 \quad (1)$$

$$a(A+B) = 1$$

$$A + B = \frac{1}{a} \quad (2)$$

$$2A = \frac{1}{a}$$

$$A = \frac{1}{2a}, \quad B = \frac{1}{2a}$$

$$\frac{1}{a^2 - x^2} = \frac{-1}{2a}\left[\frac{-1}{a-x}\right] + \frac{1}{2a}\left[\frac{1}{a+x}\right]$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{-1}{2a} \ln(a-x) + \frac{1}{2a} \ln(a+x)$$

$$= \frac{1}{2a}\left[-\ln(a-x) + \ln(a+x)\right] = \frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right) \quad \begin{matrix} x \neq a \\ a \neq 0 \end{matrix}$$

$$\star \int \frac{dx}{x^2 - 4} = -\int \frac{dx}{4 - x^2} = -\int \frac{dx}{2^2 - x^2} = \frac{-1}{4} \ln \frac{2+x}{2-x} \quad \#$$

EX 2.

Pg 24

$$\int \frac{dx}{x^3+4x}$$

$$\frac{1}{x^3+4x} = \frac{1}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

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$$A(x^2+4) + x(Bx+C) = 1$$

$$Ax^2 + 4A + Bx^2 + Cx = 1$$

$$x^2(A+B) + Cx + 4A = 1$$

Comparing The Cofficients

$$A + B = 0$$

$$C = 0$$

$$1 = 4A$$

$$A = \frac{1}{4}, \quad B = \frac{-1}{4}, \quad C = 0$$

Then $I = \int \frac{1}{x^3+4x} = \frac{1}{4}\int \frac{1}{x}\,dx - \frac{1}{4\times 2}\int \frac{2x}{x^2+4}$

$$I = \frac{1}{4}\ln x - \frac{1}{8}\ln(x^2+4) \quad \#$$

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EX6 :- 18 $\int \frac{x^5-2}{x^3+x^2}\,dx$

$$= x^2-x+1 - \frac{x^2+2}{x^2(x+1)}$$

$$\frac{1}{x+1} + \frac{2}{x^2(x+1)}$$

$$\frac{2}{x^2(x+1)} = \frac{A}{x^2} + \frac{B}{x+1} + \frac{c}{x}$$

$$\frac{2}{x^2(x+1)} = \frac{A(x+1)+Bx^2+C(x+1)x}{x^2(x+1)}$$

$2 = A(0+1)$ $\quad 2 = B(-1)^2$

$A = 2$ $\quad B = 2$

$x=1 \quad 2 = A(2) + 2(1)^2 + 2C*(1)$

$2 = 2(2) + 2(1) + 2C$

$2C + 4 = 0$

$C = -2$

$$\int \left(x^2-x+1 - \frac{3}{x+1} - \frac{2}{x^2} + \frac{2}{x}\right) dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x - 3\ln(x+1) + 2\ln x + \frac{2}{x} + C$$

$$\begin{array}{r} x^2-x+1 \\ x^3+x^2 \overline{)\, x^5 \quad\quad -2} \\ \cancel{x^5} + x^4 \\ \hline -x^4 \quad -2 \\ \cancel{-x^4} - x^3 \\ \hline x^3 \quad -2 \\ \cancel{x^3} + x^2 \\ \hline -x^2 - 2 \end{array}$$

$$Ex: \int \frac{dx}{x^2+ax+a^2}$$

$$x^2+5x+4 = (x+1)(x+4)$$

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$$x^2+3x+4 = x^2+3x+\left(\frac{3}{2}\right)^2+4-\left(\frac{3}{2}\right)^2$$
$$= \left(x+\frac{3}{2}\right)^2+4-\frac{9}{4}$$
$$= \left[x+\frac{3}{2}\right]^2+\frac{7}{4}$$
$$= \left[x+\frac{3}{2}\right]^2+\left(\frac{\sqrt{7}}{2}\right)^2$$

$$x^2+ax+a^2 = x^2+2\frac{a}{2}x+\left(\frac{a}{2}\right)^2+a^2-\left(\frac{a}{2}\right)^2$$
$$= \left(x+\frac{a}{2}\right)^2+\frac{3a^2}{4}$$
$$= \left(x+\frac{a}{2}\right)^2+\left(\frac{a\sqrt{3}}{2}\right)^2$$

$$\int \frac{dx}{x^2+ax+a^2} = \int \frac{1}{\left(x+\frac{a}{2}\right)^2+\left(\frac{a\sqrt{3}}{2}\right)^2}\,dx$$

$$\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

Let $x+\frac{a}{2}=u$

$d\left[x+\frac{a}{2}\right]=du$

$dx=du$

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$$I = \int \frac{1}{u^2 + \left(\frac{\sqrt{3}\,a}{2}\right)^2}$$

$$= \frac{1}{\frac{\sqrt{3}\,a}{2}} \operatorname{Tan}^{-1} \frac{x + \frac{a}{2}}{\frac{\sqrt{3}}{2}\,a}$$

$$= \underline{\frac{2}{\sqrt{3}\,a} \operatorname{Tan}^{-1} \frac{2\left(x + \frac{a}{2}\right)}{\sqrt{3}\,a}} \quad \#$$

E.X: 2 Pg 24: $\int \frac{x^3}{x^3 - a^3}\,dx$

$$\frac{x^3}{x^3 - a^3} = 1 + \frac{a^3}{x^3 - a^3}$$

$$\frac{a^3}{x^3 - a^3} = \frac{a^3}{(x-a)(x^2 + ax + a^2)}$$

$$= \frac{A}{(x-a)} + \frac{Bx + C}{x^2 + ax + a^2}$$

$$a^3 = A(x^2 + ax + a^2) + (Bx + C)(x - a)$$

$x = a$

$$a^3 = A(a^2 + a^2 + a^2) + 0$$

$$3a^2 A = a^3 \Rightarrow A = \frac{a}{3}$$

$x = 0$

$$a^3 = a^2 A + (0 + C)(0 - a)$$

$$a^3 = a^2 \cdot \frac{a}{3} - aC$$

$$a^3 = \frac{a^3}{3} - aC \Longrightarrow C = \frac{-2a^2}{3}$$

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Let $X = 1$

$$a^3 = A\,(1 + a + a^2) + (B + C)(1 - a)$$

$$a^3 = \frac{a}{3}(1 + a + a^2) + \left(B - \frac{2}{3a^2}\right)(1 - a)$$

$$B = \frac{-1}{3}a$$

$$\frac{a/3}{X - a} + \frac{-\frac{a}{3}X - \frac{2a^2}{3}}{X^2 + aX + a^2}$$

$$I = \int\left(\frac{-\frac{a}{3}X}{X^2 + aX + a^2} - \frac{\frac{2a^2}{3}}{X^2 + aX + a^2}\right)dX$$

$$I = -\frac{a}{3}\int\frac{X\,dX}{X^2 + aX + a^2} - \frac{2a^2}{3}\int\frac{dX}{X^2 + aX + a^2}$$

$$J = \frac{X\,dX}{X^2 + aX + a^2} \qquad X^2 + aX + a^2 \Rightarrow 2X + a$$

$$= \frac{(2X + a - a)/2\,dX}{X^2 + aX + a^2}$$

$$= \frac{1}{2}\left[\frac{2X + a}{X^2 + aX + a^2}dX - \frac{a}{2}\int\frac{dX}{X^2 + aX + a^2}\right.$$

$$= \frac{1}{2}\ln(X^2 + aX + a^2) - \frac{a}{2}$$